RESOLUTION OF THE SPACE-DRIVE ENERGY PARADOX

Todd J. Desiato

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Abstract

In a recent publication; *Reconciling a Reactionless Propulsive Drive with the First Law of Thermodynamics* [1] regarding; "A space drive", which is… "a hypothetical device that generates a propulsive force in free space using an input of power without the need for reaction mass", the author has drawn the conclusion that; "any device with a thrust-to-power ratio greater than a photon rocket would be able to operate as a perpetual motion machine of the first kind, and thus should be excluded by the First Law of Thermodynamics." In this brief note, this claim is refuted with a resolution to this *Energy Paradox*. The paradox is due to a misinterpretation of the relativistic mathematics at low velocity and resolving this issue may lead to a deeper understanding of General Relativity (GR) theory and how it applies in a low velocity, accelerated reference frame application. This exercise is an excellent example of Einstein's Equivalence Principle.

THE RESOLUTION

Presented herein is the proper interpretation of the relationship between power and force for a space drive. The problem arises when one assumes a photon rocket has a constant thrust-to-power ratio of $1/c$, [1] and can exert constant relative acceleration at low velocity, $v << c$. The author explains equation (1) as follows;

$$F = \left(\text{momentum per photon}\right) \times \left(\text{rate of photon generation}\right) = \frac{P_{in}}{c} \tag{1}$$

This equation represents the force exerted by the radiation pressure of light in free space. However, it is incorrect to use this equation to discuss the forces and momentum imparted to a massive object, which is not light in free space. Doing so results in incorrect conclusions and recommendations.

Starting from the relativistic equation governing power as a function of force and velocity;

$$P = \gamma^3 \mathbf{F} \cdot \mathbf{v} \rightarrow \gamma^3 \mathrm{F}\, v \tag{2}$$

Where, $\gamma = 1/\sqrt{1 - v^2/c^2}$ and c is the speed of light in vacuum. When applying a constant thrust-to-power ratio (N/W) the proper interpretation is that there is a *limiting velocity* which cannot be exceeded. The inverse, (W/N) gives the relativistic velocity expression,

$$\frac{P}{\mathrm{F}} = v\gamma^3 = \frac{v}{\left(1 - v^2/c^2\right)^{3/2}} = const \tag{3}$$

Where, for a <u>*constant*</u> thrust-to-power ratio of 1 N/W, the limiting velocity is 1 m/s. Therefore, it is proper to say that a specific thrust-to-power ratio will result in a specific delta-v. [2] However, it will not continue on that trajectory beyond a limited speed. At all times, energy and momentum are conserved, and Power-in = Power-out;

$$P \cdot t = mc^2\left(\gamma - 1\right), \quad E_{in} = E_{out},$$

$$P = mc^2\,\partial\gamma / dt = \gamma^3 \mathrm{v} \cdot \mathrm{F}, \quad P_{in} = P_{out}, \tag{4}$$

$$\frac{P}{\partial \mathrm{v} / \partial t} = \gamma^3 m\mathrm{v}, \quad p_{in} = p_{out}$$

The correct interpretation is such that to exceed the limiting velocity, a higher input power is required. It is incorrect to assume that because a force is being exerted, the velocity will continue to accelerate beyond the limit at which the kinetic energy exceeds the total input energy and the system becomes a perpetual motion machine of the first kind. [1]

The physics is similar in nature to *hovering* in a gravitational field, where Special Relativity does not apply. The Newtonian gravitational potential $\boldsymbol{\Phi}$ has units of $\left(m / s\right)^2$, such that the gradient derivative yields an acceleration vector. It represents the potential energy per unit mass and may be treated identically to the velocity squared in Newtonian *kinetic energy,* $\mathrm{v}^2 = 2E / m$.

In order to remain stationary at a given altitude in a gravitational field, a constant force must be exerted, along with a constant acceleration and a constant input power. There is no gain in altitude, no increase in the potential energy and no increase in velocity. Although power and force are being expended constantly.

There is however time dilation and length contraction in a gravity well at any given potential, identical to a mass that has undergone acceleration in a non-inertial reference frame to an identical potential. Therefore, a constant thrust-to-power, N/kW thruster will reach a maximum kinetic energy that is equivalent to its potential energy in a gravity well, as shown in equation (5). The potential energy per unit mass is the ratio of power/force squared.

$$\left(\frac{P}{\mathrm{F}}\right)^2 = \frac{\mathrm{v}^2}{\left(1 - \mathrm{v}^2 / c^2\right)^3} \rightarrow \boldsymbol{\Phi}_{gravity} = \frac{GM}{r} \tag{5}$$

When matter is accelerated, it is best to think of it as waves rather than particles. Suppose for example matter waves are accelerated, the wavelengths of the particle distributions are undergoing a Doppler shift, $dp = -\left(\hbar / \lambda^2\right) d\lambda$, even at relatively slow velocity. This must occur all the way down to the scale of sub-atomic particles where it is the wave functions of the nucleons that exert the most resistance to being accelerated to shorter wavelengths, and higher momentum. The wave amplitudes are simply the probability of where to find the particles but their physical behavior in this example is that of waves.

Acceleration and the Doppler shift results in matter which has been length contracted and time dilated.

$$\begin{aligned} \Delta x &\rightarrow \Delta x / \gamma, \\ \Delta t &\rightarrow \gamma \ \Delta t, \end{aligned} \tag{6}$$

This *physical* change in the properties of matter alters atomic and sub-atomic spacing, and energies, identical to how gravity contracts matter falling into a gravity well. Regardless of how miniscule the effect is at low velocity, the *relative refractive index* with respect to the vacuum refractive index is altered in the same way that a gravitational field causes gravitational lensing and alters the refractive index of the surrounding vacuum according to the potential energy per unit mass of the field.

"A constant acceleration eventually reaches a potential energy per unit mass where the power supply can no longer overcome the gradient in the Doppler shift."

Equivalently, it must overcome the gradient in the metric potential, $\Phi_{gravity}$ shown in equation (5).

This property of the matter-vacuum system may be associated with inertial mass since it is apparently the cause of relativistic mass and gravitational mass in this context.

CONCLUSIONS

There is no difference when mass is accelerated by a conventional means or by radiation pressure from a photon rocket, the solar wind, or a "space-drive". The equation for the thrust-to-power ratio is the same. A constant thrust-to-power ratio will result in a specific-delta-v, nothing more. It will not continue accelerating beyond this limiting velocity although constant power and constant force are being applied. To accelerate beyond this velocity the input power applied must be increased. At all times and for all obtainable kinetic energies, energy and momentum are conserved, and Power-in = Power-out.

This exercise has lent insight into the workings of Relativity theory by illustrating the equivalence between an accelerated reference frame and a gravitational field, as required by Einstein's Equivalence Principle.

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References

1. A. J. Higgins, *"Reconciling a Reactionless Propulsion Drive with the First Law of Thermodynamics",* arXiv:1506.00494v1, June 2, 2015.
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